RESEARCH ON PROBABILISTIC DESIGN METHOD OF FLOOD DEFENCES

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Abstract: A probabilistic design method is proposed based on reliability analysis. According to the empirical expression of code, limit state equations of overtopping, seepage stability and slope stability of the typical dike with inclination watertight facing and on two-phase fluvial facies base are formulated. The Monte Carlo method is employed to assess the stability. The influence of the geometry parameters, such as the height, slope ratio of upstream and downstream, crest width of dike, thickness and permeability coefficient of clay stratum, permeability coefficient of sand stratum, width of outer berm, width of downstream gland and thickness of downstream gland, on the reliability index of dike is discussed in detail.

Furthermore, the design process for this type of dike is suggested. Some conclusions will be helpful to the optimization of dike design.

Keywords: dike; sensitivity; reliability; design; Monte Carlo simulation

1. INTRODUCTION

Overtopping, seepage damage and land slip of dike often occur in flood period because of the effect of the shape of cross-section, properties of earth fill, geology conditions, the grain structure and property of soil, hydrological conditions, topography conditions and construction conditions. According to requirements for design and application, these hazards must be considered during the life-time of dike to satisfy requirements for seepage stability and slope stability. According to the existing code of dike design, the designing method of dike is safety margin method based on limit equilibrium analysis, which can't offer reliability index for the practical engineering. But for the probabilistic design method, the stochastic character of the geometric and strength parameters can be taken into account, so various designing indexes are adopted according to the importance of different components of dike in order to decrease engineering cost. During the past decades, the approach has improved considerably. In Netherlands, probabilistic design and risk analysis methods have been recommended by Pilarczyk K W. (1998), adopting the criterion of dikes and revetments. In China, Wang Xuancang et al. (1997), Ni Wanke (1999), Xia Wei et al. (2001) presented the probabilistic design method of slope. Considering the structure characteristics and constituent materials of flood defences, as well as the effects of various factors on instability should be considered, a relatively simple and practical design approach should be studied deeply.

In order to investigate the effects of structural parameters and geotechnical parameters on reliability indexes of overtopping, seepage stability and slope stability, a case study for typical dike on two-phase fluvial facies base is performed by adopted Monte Carlo method and probabilistic designing method of dike is proposed. This will be helpful to fulfill the existing designing code of dike and to realize the transition of the designing method of dike from traditional method to probabilistic method together with the reliability study of outlet structure, cut-off walls and wake walls.

2. ANALYSIS PROCEDURE OF RELIABILITY

The main analysis methods of reliability for engineering structures are the checking point method and Monte Carlo method. The checking point method is based on expansion in a Taylor series at checking point and simplification from non-linear function to liner function. So it has a low accuracy and bad convergence. Monte Carlo method calculates the reliability degree through many random samplings. So it can be applied for reliability analysis of flood defence with non-liner limited state function. The theory of the method recommended by Huang Kezhong, Mao Shanpei (1987) is as following:

Method of Random Sampling

Firstly, normally distributed random numbers are obtained on interval of (0,1) by mixed congruence method and multiplicative congruential method. The recursion formula of mixed congruence method is:

$$\begin{aligned} x_i &= (\lambda x_{i-1} + C)(\bmod M) \\ r_i &= x_i / M \, (i = 1,2,\cdots,n) \end{aligned} \tag{1}$$

Where, λ、 x_0、 C and M are constants chosen. Formula (1) shows the remainder of $\lambda x_{i-1} + C$ divided by M is x_i and random numbers r_i on interval of (0,1) are obtained by x_i divided by M .

The random numbers sequence $\{r_i\}$ is transferred to normally distributed random numbers sequence $\{R_i\}$on interval of (a,b).

$$R_i = a + (b - a)r_i \tag{2}$$

By method of inverse function, normally distributed random numbers $\{r_i\}$ are transferred to random numbers in order to meet a certain referred probability distribution. The premise of it is the inverse function of its empirical distribution exists. Otherwise the random variable function method will be adopted.

Suppose X is a continuous random variable whose distribution function is $F_X(x)$ and whose inverse function exists. r is the value of random variable R whose distribution type is uniform distribution function is $F_R(r)$. If the cumulative probability $F_X(x) = r$ is given the following can be drawn:

$$x = F_X^{-1}(r) \tag{3}$$

If $\{r_i\}$ has been known sequence of random numbers meet $F_X(x)$ will be obtained:

$$x_i = F_X^{-1}(r_i) \quad (i = 1,2,\cdots,n) \tag{4}$$

Basic Process of Monte Carlo Method

(1) Probabilistic distribution models and distribution parameters of the variables related to reliability analysis are determined;

(2) The first random sampling of all variables is done, and the result is used in the reliability function;

(3) Repeat random sampling independently for the total number of simulations n, and then failure probability is estimated.

Results and Accuracy of Monte Carlo Method

In reliability analysis for engineering structures, the limit state function is $Z = g(x_1, x_2, \cdots, x_n)$ and the failure probability is:

$$P_{\mathrm{f}} = P(g(x_1, x_2, \cdots, x_n) \le 0) \tag{5}$$

When basic variables are assigned values by random sampling, the result is $g(\cdot) > 0$ or $g(\cdot) \le 0$. So index function can be defined as following:

$$I(g(x_1,x_2,\cdots,x_n)) = \begin{cases} 1, & g(\cdot) \le 0 \\ 0, & g(\cdot) > 0 \end{cases} \quad (6)$$

According to Bernoulli's theorem and characteristics of normally distributed random variable the failure probability is:

$$\hat{P}_{\mathrm{f}} = \frac{1}{N}\sum I(g(x_1,x_2,\cdots,x_n) \le 0) = \frac{M}{N} \quad (7)$$

Where, M is the total number that $g(\cdot) \le 0$ in the total number N of simulations.

Formula (7) is not the only formula to calculate failure probability. The distribution of function is fit according to simulating result and the first moment $\hat{\mu}_x$ and the second moment $\overline{\sigma}_x^2$ are obtained. Therefore, the reliability index β can be obtained as following:

$$\beta = \frac{\overline{\mu}_x}{\overline{\sigma}_x} \quad (8)$$

The error of Monte Carlo method is expressed by $\hat{\mu}_x$. The more discrete the function value Z is, the larger the error is. When simulating number is sufficiently large the standard deviation of estimation values obtained by simulating sample inverses with the square root of simulating number. So the accuracy increases with the increase of simulating number. In general, when simulating number is more than $N \ge 100/P_{\mathrm{f}}$ the accuracy may be satisfactory.

3. PROBABILISTIC DESIGN OF A FLOOD DEFENCE

An idealized flood defence with inclination clay layer is analyzed, as shown in Fig 1. It is supposed to be a two-phase fluvial facies with its basement constituted of weak pervious clay stratum and strong pervious sand stratum.

The values of some design variables and deterministic parameters and random variables in this analysis are listed in Table 1.

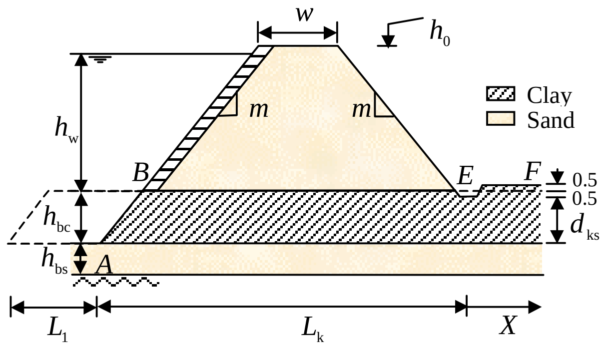

Fig. 1 Typical dike cross section with inclination facing on two-phase fluvial facies basement

Table 1 Variables of this study

Variable	Description	Unit	Type	Mean value	Standard deviation
h_0	Height	m	Design variable	11.0	
h_{bc}	Thickness of clay	m	Design variable	3.5	
L_1	Width of foreland	m	Design variable	10.0	
L_k	Effective seepage path	m	Design variable	68.75	
m	Slope ratio		Design variable	2.5	
w	Crest width	m	Design variable	5.0	
t_{sb}	Thickness of permeable gland	m	Design variable	0	
h_{bs}	Thickness of sand	m	Deterministic	2.5	
γ_{nk}	Buoyant unit weight of clay	kN/m^2	Deterministic	9.0	
γ_w	Unit weight of water	kN/m^2	Deterministic	10.0	
γ_{sb}	Bulk gravity of permeable gland	kN/m^2	Deterministic	18.0	

d_{ks}	Effective thickness of clay	m	Normal	3.5	0.7
h_w	Flood water level	m	Exponent	8.34	0.9
k_c	Permeability coefficients of clay	m/s	Normal	10^{-8}	$2*10^{-8}$
k_s	Permeability coefficients of sand	m/s	Normal	10^{-5}	$0.75*10^{-5}$

3.1 Overtopping

The water level is the deterministic factor in the determination of the height of crest. The reliability function of overtopping is:

$$z_1 = h_0 - h_w - h_s - e \quad (9)$$

Where, h_s is the swash height; e is the surge height.

There are many factors which have effects on the height of dike crest. The reserved settlement is set in design because of the consideration of construction precision and consolidation settlement after completion. Suppose h_0 is normally distributed, and the probability which the reserved settlement is more than 0.1m is less than 2%. Therefore, according to the expression of normally distribution, the following can be drawn:

$$\frac{h_d - h_0}{\sigma_R} = 1.96 \quad (10)$$

where, h_d is the designing height of dike. Then formula (10) can be written as:

$$\sigma_{h_0} = 0.1/1.96 = 0.051 \quad (11)$$

The maximum height of flood level h_w is supposed to be exponential distribution with $\mu_{h_w} = 8.34$m and $\sigma_{h_w} = 0.9$ according to general experience. The swash height h_s is supposed to be normal distribution with $\delta_{h_s} = 0.69$.

The average wave height μ_h is recommended by Chinese Standard (1998):

$$\frac{g\mu_h}{V^2}=0.13\mathrm{th}\left[0.7\left\{\frac{gh_{\mathrm{a}}}{V^2}\right\}^{0.7}\right]\times\mathrm{th}\left\{0.0018\left[\frac{\mathrm{g}F}{V^2}\right]^{0.45}\left[0.13\mathrm{th}\left[0.7\frac{gh_{\mathrm{a}}}{V^2}\right]^{0.7}\right]^{-1}\right\} \qquad (12)$$

Where, μ_h is the average wave height, m; V is the calculating wind speed, the value of it is 18m/s; F is the length of wind field, the value of it is 2000m; h_{a} is the average water depth of the water area, the value of it is 10.5m.

Swash height is

$$\mu_{h_s}=\frac{K_\Delta K_V\sqrt{\mu_h\mu_\lambda}}{\sqrt{1+m^2}} \qquad (13)$$

Where, $m=2.5$; $K_\Delta=0.75\text{-}1.0$, 0.85 can be used; $K_V=1.0\text{-}1.3$, 1.1 can be used; μ_λ is the wave length.

$$\mu_\lambda=\frac{g{\mu_T}^2}{2\pi}\mathrm{th}\frac{2\pi h_{\mathrm{a}}}{\mu_\lambda} \qquad (14)$$

Where, μ_T is the average wave period, and $\mu_T=4\sqrt{\mu_h}$.

Surge height is

$$e=\frac{KV^2F}{2gh_{\mathrm{a}}}\cos\beta\approx\frac{KV^2F}{2gh_{\mathrm{a}}} \qquad (15)$$

Where, the angle of wind direction β is 0^0; K is the combined coefficient of friction resistance, and its value is 3.6×10^{-6} .

The following can be obtained: $\mu_{h_s}=0.638$; $\sigma_{h_s}=\delta_{h_s}\cdot\mu_{h_s}=0.69\times0.638=0.4395$.

Certain extra-height should be set in practical engineering according to the grade of dike, when the observation data in hydrograph analysis can not obtained. But it is not considered in this study.

Therefore, reliability index β_1 by Monte Carlo method corresponding to the mean value of the height of dike μ_{h_0} is obtained as shown in Fig .2. It shows when μ_{h_0} is 11.0m, β_1 is 2.363, and P_f is 0.914%.

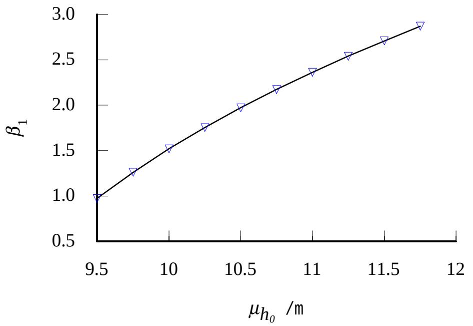

Fig. 2 Relationship of height of dike and reliability index

3.2 Piping

The inclination facing is supposed to have good impervious property and the type of seepage damage is piping.

Under high water level, piping will take place at vulnerable area of clay layer because it can be penetrated by artesian head, and then a breach is developed. By mechanical equilibrium condition, the reliability function of piping z_2 is obtained by by Chinese Standard (1998):

$$z_2 = \gamma_{nk} d_{ks} - \gamma_w h_{ap} + \gamma_{sb} t_{sb} \tag{16}$$

Where, h_{ap} is the residual head of lower bound of the weak permeable stratum which can be calculated:

$$h_{ap} = \frac{h_w}{1 + A * L_k + \text{th} A * L_1} e^{-AX} \tag{17a}$$

$$A = \sqrt{\frac{k_c}{k_s h_{bc} h_{bs}}} \tag{17b}$$

Where, A is a coefficient. Because of natural and manmade factors, the thickness and impervious strength of clay layer are non-homogeneous and pond and low land often occur at inner side of dike. Therefore, the effective thickness of clay d_{ks} below dike canal is taken as random variable and normally distributed with mean value 3.5m and standard deviation 0.7, as shown in Table 1. X is the distance to point E in Fig.1.

Formulae (17) show that reliability index has something to do with flood water head, property and density of fill material of dike body, structure and property of soil layer of dike foundation, width of outer berm, gland behind dike, etc. Adopting Monte Carlo method sensibility analysis of these parameters is performed as following. In sensibility analysis only one of the parameters varies, others remain values of Table 1.

Thickness of clay layer

The relationship of thickness of clay layer and reliability index of seepage stability β_2 is shown in Fig.3. It shows that thickness of clay layer has strong influence on seepage stability of dike foundation.. The increase of β_2 with the increase of thickness of clay layer can be given.

Width of foreland

Fig.4 shows that the increase of β_2 with the increase in width of foreland. The reason is that the increase of width of seepage path of foundation intensify the capability of impervious of soil. But in this example, β_2 decrease slightly when the width of foreland increases to more than 20.0m. It is an effective method to control seepage for the structure to adopt natural covering when there is wide foreland at upstream in practical engineering.

Slope ratio of flood defence

Fig.5 shows that β_2 increase with the increase of slope ratio of flood defence.

Width cross dike crest

It is shown in Fig.6 that β_2 increase linearly with the increase of width cross dike crest. The increase of slope ratio and width cross dike crest changes the width of seepage path. But it will raise the construction cost. It is an important problem to be deeply studied to balance the relationship of construction cost and expected flood damage with the introduction of conception of economic assessment.

Permeability coefficient of clay layer

The permeability coefficient of soil can reflect mechanical composition, structure, tightness and pore size. The decrease of β_2 with the increase of permeability coefficient of clay layer is shown in Fig.7.

When the permeability coefficient of clay layer is more than 0.5×10^{-8}, β_2 decreases obviously.

Permeability coefficient of sand layer

The decrease of β_2 linearly with the increase of permeability coefficient of sand layer is shown in Fig.8. The conclusion can be drawn embankment fill should be chosen carefully and compaction quality should be controlled strictly.

It can be shown that seepage stability can be satisfied with the typical values in Table 1 from this study. The influence of these factors on seepage stability should be considered completely, and reasonable selections of parameters should be made prudently. For example, when the thickness of clay layer is not enough big or the clayey can not obtained in site, some measures such as widen crest and slope ratio can be taken.

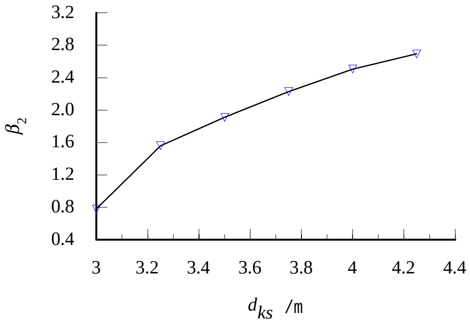

Fig 3 Thickness of clay stratum

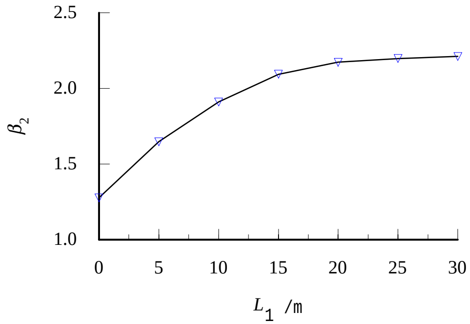

Fig. 4 Width of foreland

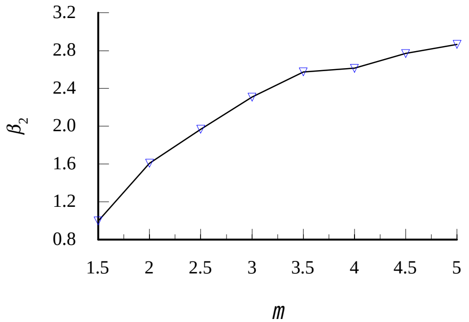

Fig 5 Slope ratio

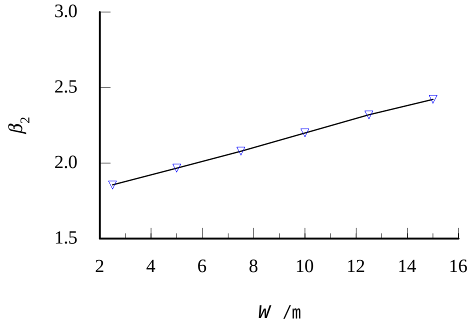

Fig. 6 Width across the crest

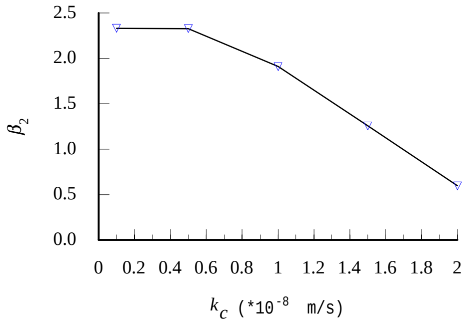

Fig 7 Permeability coefficient of clay stratum

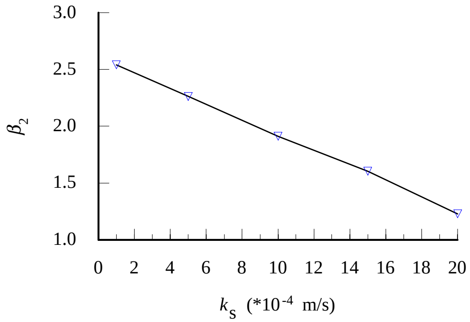

Fig. 8 Permeability coefficient of sand stratum

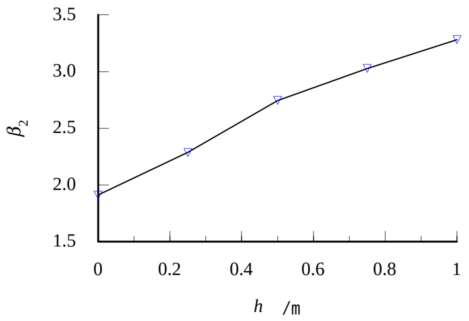

Fig. 9 Thickness of gland

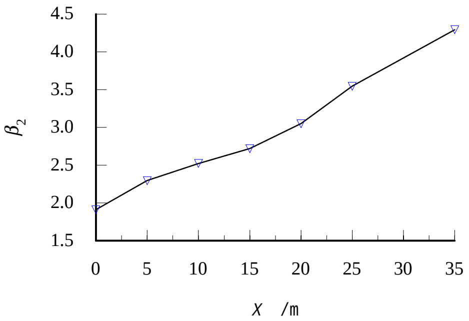

Fig. 10 Width of gland

3.3 Slope stability of dike

The minimum critical factor of safety against sliding and dangerous slip surface are obtained according to simplified Bishop method. The factor of safety is:

$$F = M_{\mathrm{r}} / M_{\mathrm{o}} \tag{18}$$

Where, M_{r} 、 M_{o} are moment against sliding and sliding moment respectively. The reliability function of slope stability of dike z_3 is:

$$z_3 = F - 1 \tag{19}$$

By Monte Carlo method numerical simulation for formula (19) is performed and statistics of some geotechnical parameters are shown in Table 2. The relationship of slope ratio of dike and reliability index of slope stability is shown in Fig.11. It shows that when slope ratio is 2.5, β_3 is 2.756, and the corresponding failure probability is 0.289%. In addition, the probability of slope instability is lower than that of overtopping and piping, because of the existence of outer sloping watertight layer.

Furthermore, factors of safety of different slope ratios are also shown in Fig.11. The minimum critical factor of safety in convention design is assumed to be K_0 =1.5, the corresponding reliability index is $\beta_3 = 2.63$.

3.4 Probabilistic Design Process of Dike

Based on the above-mentioned analysis, the probabilistic design process of dike is as following:

The height of crest of dike is determined by analysis for data of flood water level and wind wave.

The thickness of clay layer of dike is determined by characteristics of piping.

The slope ratio of dike, width cross crest of dike, width of foreland, width of gland, thickness of gland and seepage characteristics of fill are determined by characteristics of piping. Traffic demand during flood should be considered in design of width cross crest of dike.

Strength characteristics of soil are determined according to slope stability analysis.

The revetment, drainage facility of slope surfaces, impervious facility and drainage facility are set according to actual need.

Table 2 Statistic of geotechnic parameters

Type	Symbol	Name/unit	Distribution	Mean value	Standard deviation
Sand	ϕ	Friction angle (0)	Normal	35.0	3.5
Clay	c	Cohesive (kPa)	Normal	10.0	2.0
	ϕ	Friction angle (0)	Normal	20.0	4.0

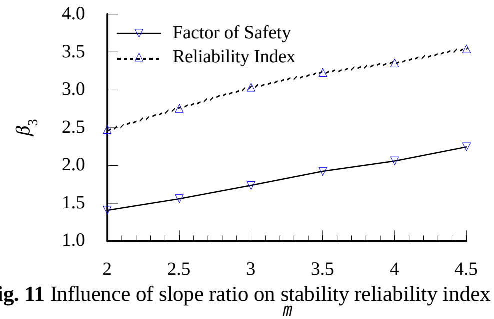

Fig. 11 Influence of slope ratio on stability reliability index

4. CONCLUSIONS

Parametric sensibility study has been performed on typical dike. The parameters investigated are the structure design parameters of dike and seepage coefficient of soil. The following conclusions can be obtained: (1) The reliability index of overtopping increases with the increase of height of dike. (2) The reliability index of seepage stability of dike foundation increases with the increase of slope ratio of dike, width cross crest of dike, width of clay layer of dike foundation, width of foreland, width and thickness of gland behind dike. (3) The reliability index of seepage stability of dike foundation decreases with the increase of seepage coefficient of clay layer and sand layer of dike foundation. (4) The reliability index of slope stability increases with the increase of slope ratio of upstream and downstream of dike.

This problem should be deeply studied to determine object reliability and to optimize the design of dike considering geometry constitution characteristics of dike structure, construction cost and damage of dike.

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