

# Structure Risk Analysis with Slope Stability and Seepage of Dike at High Flood Level

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**Abstract:** Based on the mathematical model proposed by Wang et al, the Monte Carlo method is respectively employed to assess the reliability of stability of downstream slope and seepage of the dike at high flood level. The influence of the geometry parameters, such as slope of upstream and downstream, crest width, height of dike etc, on the risk of dike is discussed in detail. In addition, sensitivity analysis of statistical characteristic variables of shear strength parameters is conducted to discuss the influences of soil cohesion and friction angle on the reliability index. This will be helpful to the optimization of dike design.

**Key words:** dike; sensitivity; reliability; risk; Monte Carlo simulation

## 1. INTRODUCTION

Recently, drastic floods have occurred in some large river basins. Such as overflowing, leakage, immersion and breakage will take place when dikes are at high water level, so it is practical to proceed risk analysis for slope instability and seepage deformation instability of flood defenses. Besides, dikes must be heightened and widened after flood. Therefore, it is important to study the influence of the variation of structural forms of flood defense on structural risk for design, construction and safety management of dikes.

In Netherlands, some methods of probabilistic design and safety assessment of embankment and revetment engineering have been discussed by some design guides and handbooks, such as Report 141 (1990). Combining typical dike of Nanjing section of Yangzi River dikes, mathematical model

for calculating structural risk of flood defenses is proposed by Wang Zhuofu et al. (1998). In this paper, the influence of variability of geotechnic statistic parameters and variation of geometry of dikes on structural risk will be discussed, which is helpful to offer some theoretical bases for forecasting dike potential dangers.

## 2. FAILURE MODES AND MATHEMATICAL MODEL OF STRUCTURAL RISK OF DYKES

According to data sorting of dike instability damage, principal failure modes of dike instability mainly include slope instability and seepage deformation instability. Assuming the both of them are completely independent, risk degrees of slope instability  $R_s$  and seepage deformation instability  $R_p$  of dikes can be studied separately, and then the total risk  $R$  degree of dikes can be obtained by Wang Zhuofu et al. (1998):

$$R = R_s + R_p \quad (1)$$

### 2.1 Calculating model for instability risk of flood defenses

Based on limiting equilibrium theory of soil mechanics, the reason for slope instability of flood defenses is that the sliding moment  $M_s$  exceed the resisting moment  $M_r$ . Therefore, the mathematic model for calculating the risk degree of slope instability of dikes can be expressed as following:

$$R_s = P(M_s > M_r) = \int_{M_r}^{\infty} f(M_s) dM_s \quad (2)$$

where,  $f(M_s)$  is the function of probability density distribution of dike sliding moment. Obviously, it is difficult to calculate directly adopting formula (2). But it can be discreted in practice:

$$R_s = \sum_{i=1}^N F_S(H_{is}) \cdot \Delta F_0(H_{is}) \quad (3)$$

where,  $F_S(H_{is}) = \int_{M_r}^{\infty} f(M_s / H) dM_s$  is the probability that sliding moment  $M_s$  exceed moment against sliding  $M_r$  at specified flood water level;  $\Delta F_0(H_{is})$  is the probability of segment  $i$  the frequency exceedance curves of water levels;  $N$  is the number of segment of curve of flood water level and frequency that need to be calculated.

Based on the soil seepage theory, the reason for seepage instability deformation (such as piping

or sand boil) is that seepage gradient  $J$  of soil exceed critical hydraulical gradient  $J_c$ , that is  $J > J_c$ . So the mathematical model for calculating risk degree of seepage deformation instability of flood defences is expressed as:

$$R_p = P(J > J_c) = \int_{J_c}^{\infty} f(J)dJ \quad (4)$$

where,  $f(J)$  is the function of probability density distribution of seepage gradient. Similarly, it is difficult to solve by formula (4), because  $f(J)$  is in relation to soil properties, structure of dikes and flood water level. The discrete formula for calculating  $R_p$  can be obtained by adopting the same method as  $R_s$ :

$$R_p = \sum_{i=1}^N F_J(H_{is}) \cdot \Delta F_0(H_{is}) \quad (5)$$

In which  $F_J(H_{is}) = \int_{J_c}^{\infty} f(J/H)dJ$  is the probability of seepage gradient  $J$  exceed critical gradient  $J_c$  when  $H_{is}$  is specified.

## 2.2 Calculations for failure probability $F_S(H_{is})$ and $F_J(H_{is})$

### 2.2.1 Failure probability $F_S(H_{is})$

In general, analytic solution of integration of  $F_S(H_{is})$  in formula (3) can not be obtained easily, because  $f(M_s/H)$  in  $\int_{M_r}^{\infty} f(M_s/H)dM_s$  is a very complex function. But which can be solved by Monte Carlo method easily. The main calculating procedures are as below:

- (1) The minimum safety factor of slope sliding of dikes and the dangerous sliding surface are determined by adopting the simplified Bishop method;
- (2) Producing the pseudo random numbers, random sampling for geotechnic parameters  $c$ 、 $\phi$  and  $\gamma$  is performed;
- (3) Calculating the sliding moment  $M_s$  and moment against sliding  $M_r$  for given sliding circle;
- (4) Counting the number that  $M_s > M_r$  and the number is recorded as  $m$ ;

(5) Repeating from steps (2) to (4) for  $n$  times until convergence has been attained (in this paper  $n=100,000$ );

(6) According to Bernoulli's theorem and characteristics of normally distributed random variable,  $F_S(H_{is}) = m/n$  can be obtained.

### 2.2.2 Failure probability $F_J(H_{is})$

The height of seepage exit of downstream slope of the dike under various water levels is determined using the method proposed by the code[0], and then the seepage path is estimated. The seepage gradients under various water levels are obtained.

According to theory and tests of soil mechanics, the critical seepage gradient of the soil depends on many factors, such as grain diameter, size grading, structure, porosity, bulk gravity and quality of construction, which are random variation. To simplify the analysis, only the variability of the critical seepage gradient of the soil is considered. Assuming the critical seepage gradient of the soil to be normally distributed, then the probability that seepage gradient  $J$  exceed critical gradient  $J_c$  at a certain water level is calculated by Monte Carlo method.

Based on Visual Fortran and Visual Basic, the program which can calculate seepage gradient and failure plane of different modes of dikes with different water levels and structure shape is developed.

## 3. SENSIBILITY ANALYSIS OF STATISTICAL VARIATION

As a typical flood defence an example, the upstream slope ratio is  $m_1=3$ , the downstream slope ratio is  $m_2=3$ , the width across the crest is  $w=7\text{m}$ , the height of the dike is  $h_0=10\text{m}$ , the water level of upstream is  $H_{uw}$ , the water level of downstream is  $H_{dw}$ . In reliability analysis, many variables, such as shear strength parameters, bulk gravity, pore water pressure, critical gradient, water level of upstream and downstream should be taken to be random variables. To simplify, some geotechnic random parameters used in this study are listed in Table 1. Note that reliability index  $\beta$  reflects risk degree directly. Therefore, the following will study the influence of the variation of statistics of geotechnic parameters on reliability index  $\beta$ . In each case only one parameter vary, other parameters keep the values as listed in Table 1.

**Table 1** Statistic of geotechnic parameters

Stochastic variables	Symbol	Name/unit	Distribution type	Mean value	Standard deviation
$x_1$	$c$	Cohensive (kPa)	Normal	12.54	2.8
$x_2$	$\phi$	Inner friction angle(0)	Normal	21.58	3.5
$x_3$	$\gamma$	Bulk gravity (kN/m <sup>3</sup> )	Normal	18.84	3.1
$x_4$	$J_c$	Critical seepage gradient	Normal	0.55	0.093

### (1) Influence of mean values of $c$ 、 $\phi$ on $\beta_s$

Relationship curves of  $\mu_c \sim \beta_s$  and  $\mu_\phi \sim \beta_s$  are illustrated in Fig.1 and Fig.2, respectively.  $\beta_s$  will vary with the variation of mean value of  $c$  or  $\phi$ , when other parameters keep constant. For example, when  $\mu_c$  increases from 8.54kPa to 16.54kPa,  $\beta_s$  will increase from 2.455 to 5.282. When  $\mu_\phi$  increases from 15.580 to 27.580,  $\beta_s$  will decrease from 4.015 to 3.737. Thus, it can conclude that the variation of mean values of  $c$ 、 $\phi$  have different influence on reliability index  $\beta_s$ ,  $\mu_c$  is more sensitive.

### (2) Influence of coefficients of variability of $c$ 、 $\phi$ on $\beta_s$

Fig.3 and Fig.4 show that the relationship curves of  $\delta_c \sim \beta_s$  and  $\delta_\phi \sim \beta_s$ . It can be seen that  $\beta_s$  will vary with the variation of coefficients of variability of  $c$  or  $\phi$ . For example, when  $\delta_c$  decreases from 3.218 to 2.418,  $\beta_s$  will increase from 3.391 to 4.501. When  $\delta_\phi$  decreases from 5.52 to 1.52,  $\beta_s$  will increase from 3.845 to 3.881. It is shown that the variation of coefficients of variability of  $c$ 、 $\phi$  also have different influence on reliability index  $\beta_s$ ,  $\delta_c$  is more sensitive.

In addition, the variation of mean value and standard deviation of bulk gravity of the soil have little influence on risk degree of structures. So bulk gravity can be taken as a deterministic parameter in risk analysis.

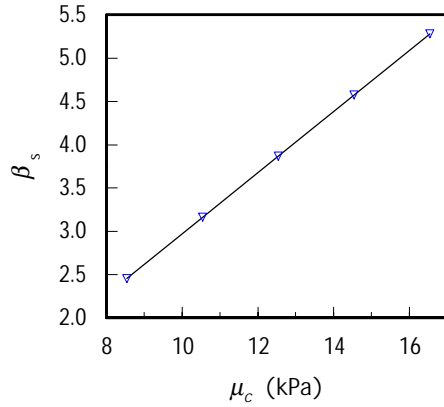


Fig.1 Relation between  $\mu_c$  and  $\beta_s$

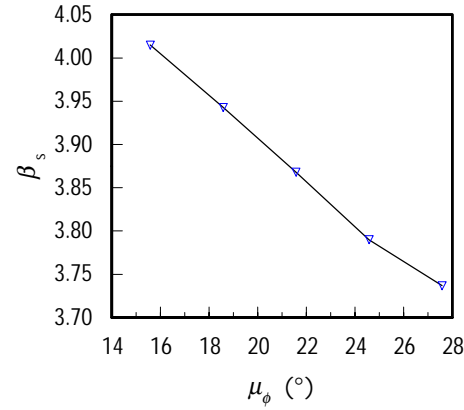


Fig.2 Relation between  $\mu_\phi$  and  $\beta_s$

**(3) Influence of mean value and coefficient of variability of critical seepage gradient  $J_c$  on  $\beta_p$**

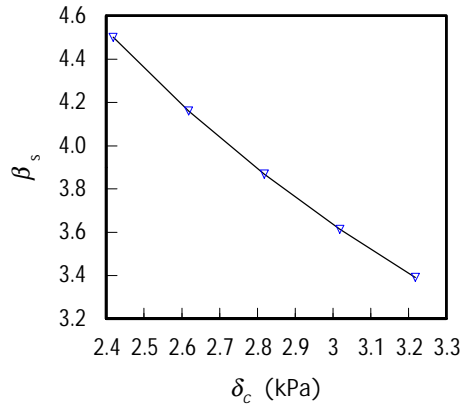
Relationship curves of  $\mu_j \sim \beta_p$  and  $\delta_j \sim \beta_p$  are shown in Fig.5 and Fig.6. When  $\mu_j$  increase from 0.45 to 0.65, the value of  $\beta_p$  increase from 2.849 to 5. When  $\delta_j$  increase from 0.073 to 0.123, the value of  $\beta_p$  decrease from 5 to 2.967. It can be concluded that statistics of critical seepage gradient  $J_c$  strongly influence the reliability index  $\beta_p$ . Due to the critical seepage gradient is determined by the constitution and structure, coefficient of inhomogeneous, bulk gravity and construction, so the choice of soil type and quality-controlling of construction rationally are very important.

**Table 2** Reliability indexes of  $\beta_s$  and  $\beta_p$  with different distribution type

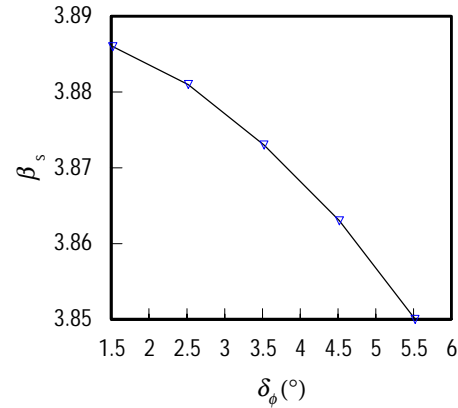
Index	Normal	Log-normal	Extremal type I largest
$\beta_s$	3.868	4.911	5.476
$\beta_p$	3.925	4.405	5.32

#### (4) Influence of distribution type of $c$ 、 $\phi$ 、 $J_c$ on $\beta$

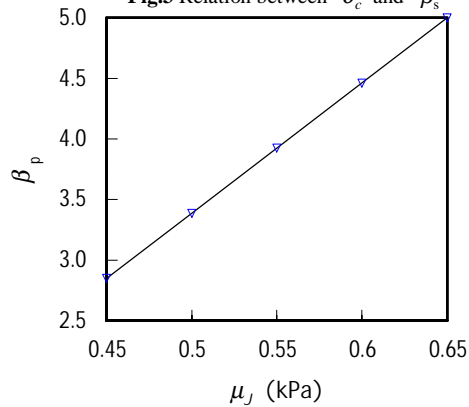
The reliability indexes of the above-mentioned example with shear strength index  $c$ 、 $\phi$  and critical gradient  $J_c$  have different distribution types. They are assumed and shown in Table 2. The conclusion can be drawn that the reliability index is smallest when the variable is normally distributed. Therefore, it is more safety to suppose the variable is normally distributed in this study.



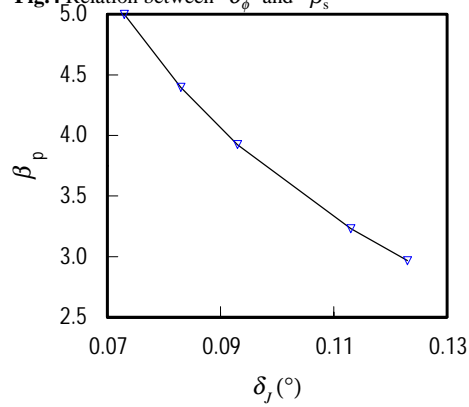
**Fig.3** Relation between  $\delta_c$  and  $\beta_s$



**Fig.4** Relation between  $\delta_\phi$  and  $\beta_s$



**Fig.5** Relation between  $\mu_J$  and  $\beta_p$



**Fig.6** Relation between  $\delta_J$  and  $\beta_p$

#### **4. INFLUENCE OF GEOMETRY OF DYKES ON STRUCTURAL RISK**

In order to investigate the effects of geometry of dikes on the structural risk at a certain water level (0.5 meter below the crest height), different values of slope ratio, width across the crest and height of the dike is performed. The following results are obtained with varying only one of the parameters. It is shown that the instability risk of downstream slope is much larger than that of upstream slope, only the risk of downstream slope is considered.

##### **4.1 Upstream slope ratio**

The risk degrees of slope instability and seepage deformation instability of the dike with different upstream slope ratios are shown in Fig.7. It can be seen that the variation of upstream slope ratio has little influence on the stability of downstream slope. The risk of piping obviously decreases with the increasing of upstream slope ratio, because of the increasing of the length of seepage path.

##### **4.2 Downstream slope ratio**

Fig.8 shows the risk degrees of different downstream slope ratios. The risk degrees of slope instability and seepage deformation instability decrease drastically with the increase of downstream slope ratio. It should be noted that more flat slope and platform can improve the capability of the dike body to resist seepage damage in practical engineering, which is rational in theoretical and the costs will increase.

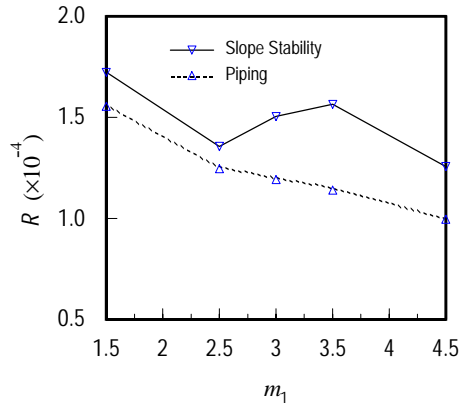
##### **4.3 Width cross the crest**

The instability risks of flood defences with different width cross the crest are shown in Fig.9. The variation of  $w$  has little influence on the risk of slope instability, but has obvious influence on the risk of piping.

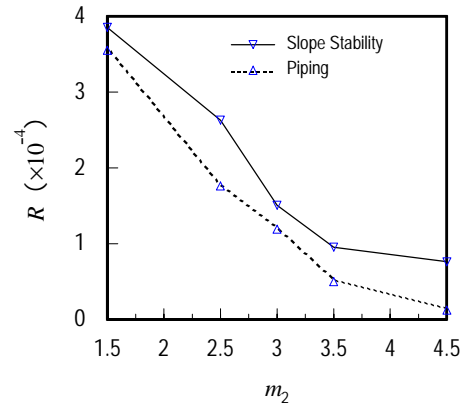
##### **4.4 Height of the dike**

Fig.10 shows the instability risks with different heights of the dike. The conclusion can be drawn that the risks of slope instability and seepage deformation instability increase with the increasing of height of the dike. There are two reasons for the increasing of risk of slope instability. One is the water level and saturation line increase with the increasing of height of the dike. The other is the increasing of sliding moment is more quickly than the increasing of moment against sliding. The reason for the increasing of risk of seepage deformation instability is the increasing of the height of the seepage exit of downstream slope is larger than the increasing of the water level of upstream, then the seepage gradient increases.

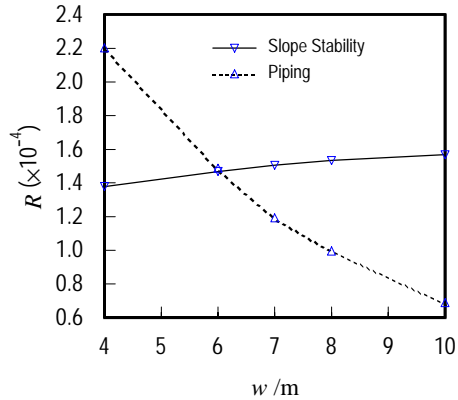




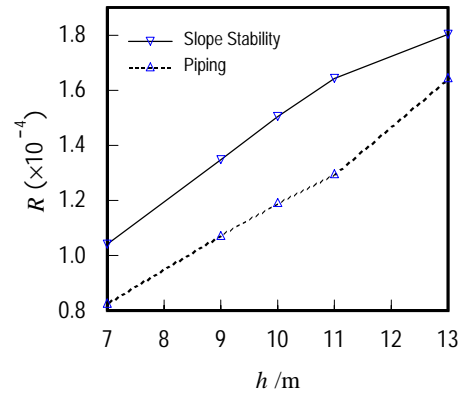
**Fig.7** Influence of upper slope ratio



**Fig.8** Influence of down slope ratio



**Fig.9** Influence of crest width



**Fig.10** Influence of height of dike

The above results of risk analysis for slope instability and seepage deformation instability are obtained at a certain water level under the condition of steady flow. In fact, a certain water level occurs with certain frequency as shown in Fig.11. Flood defences are often at various water levels, the risk of the typical cross-section with water level from 8.0m to 10.0m is 0.328%, as shown in table 3. The safety grade of the dike can be evaluated according to an allowable risk degree.

Based on the above-mentioned theory, the safety assessment software system on dike has been

developed by using Visual basic and database, some computing results of whole dike sections can be real-time displayed. As shown in Fig.12, this system has been applied successfully to the modern management on dike of a city.

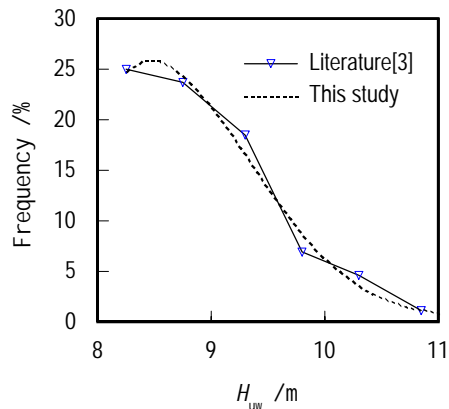


Fig.11 Frequency exceedance curves



Fig.12 Risk evaluation system on whole dike sections of a city

Table 3 Structure risk at various water levels  $H_{uw}$

$H_{uw}$ (m)	Slope instability		Seepage instability		Total
	$\beta$	$R_s$ ( $\times 10^{-4}$ )	$\beta$	$R_p$ ( $\times 10^{-4}$ )	$R$ ( $\times 10^{-4}$ )
8.0	3.89	11.64	4.35	1.59	13.23
8.5	3.88	7.97	4.26	1.88	9.85
9	3.87	3.97	4.07	1.74	5.71
9.5	3.86	1.51	3.92	1.19	2.70
10	3.85	0.51	3.75	0.76	1.27
Sum		25.61		7.16	32.77

## 5. CONCLUSIONS

The statistics parameters of shear strength index and distribution types of  $c$ 、 $\phi$  have some influence on the reliability index of slope instability.  $\mu_\phi$  is more sensitive than  $\mu_c$ ,  $\delta_\phi$  is more

sensitive than  $\delta_c$ .  $\beta_s$  gains the minim value when  $c$  and  $\phi$  are taken to normally distributed. Besides, the statistics parameters of seepage gradient have obvious influence on the reliability index of seepage deformation instability.

The variation of the geometry of dikes has effects on  $\beta_s$  and  $\beta_p$ . The variation of upstream and downstream slope ratio only has effects on the corresponding slope instability risk. Seepage path prolongs with the increasing of slope ratio, and the risk of piping decreases. Increasing of width cross the crest has great influence on the risk of seepage deformation instability and has little influence on the risk of slope instability. The risks of slope instability and seepage deformation instability increase with the increasing of the height of the dike.

To make optimum design for flood defences considering the structure shape, engineering cost, failure modes and their influence factors, more efforts should be paid on this challenging topic.

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